Mini-course 3

Uncertainty and sensitivity analysis

Dr Raeesa Manjoo-Docrat

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- Introduction

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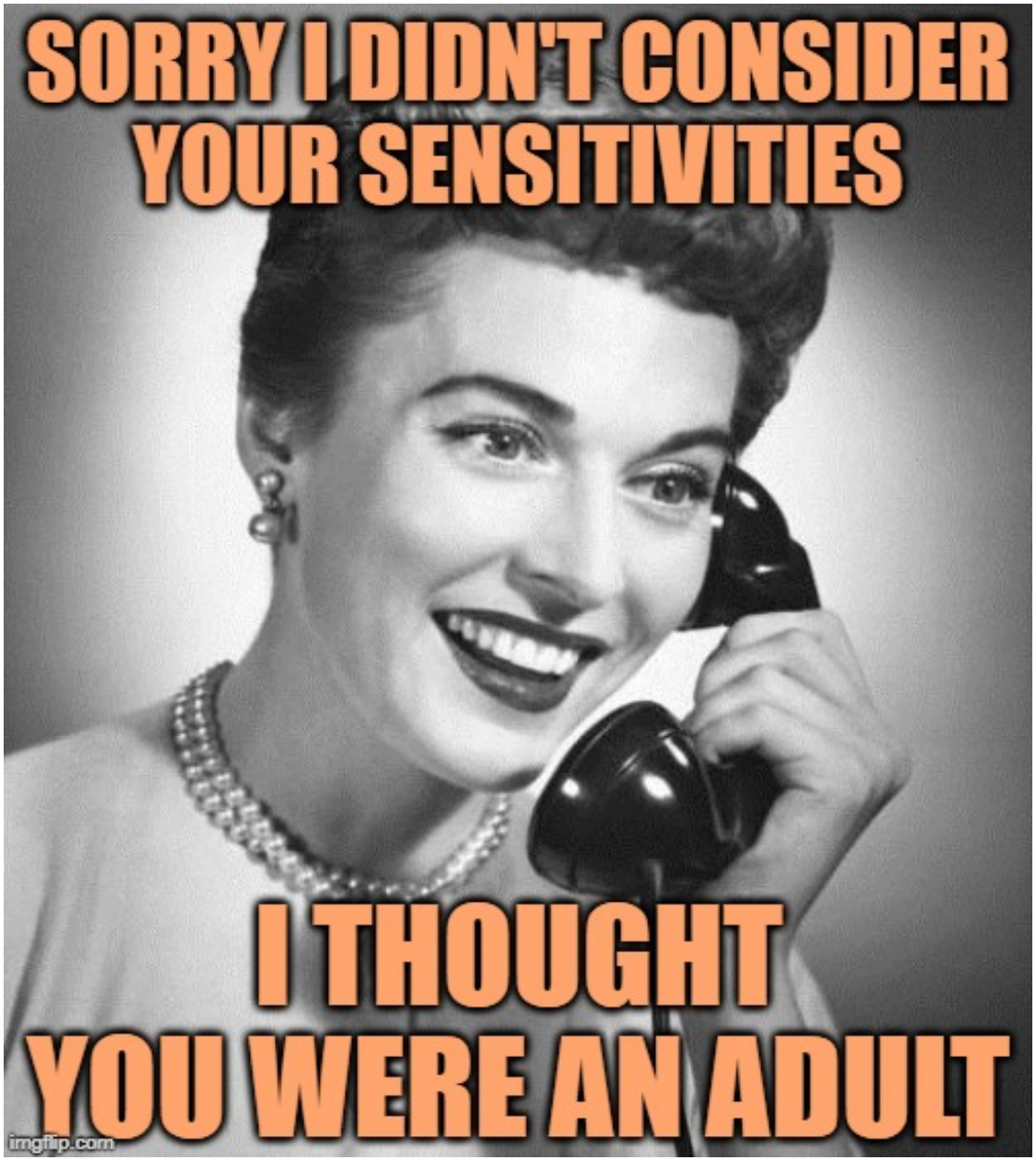
- Discussion


SORRY I DIDN'T CONSIDER YOUR SENSITIVITIES
I THOUGHT YOU WERE AN ADULT
imgflip.com

Introduction

> Uncertainty and sensitivity analysis are techniques used to understand how changes in the input parameters of a model impact the output.

It helps in making more informed decisions by assessing the robustness and reliability of models.

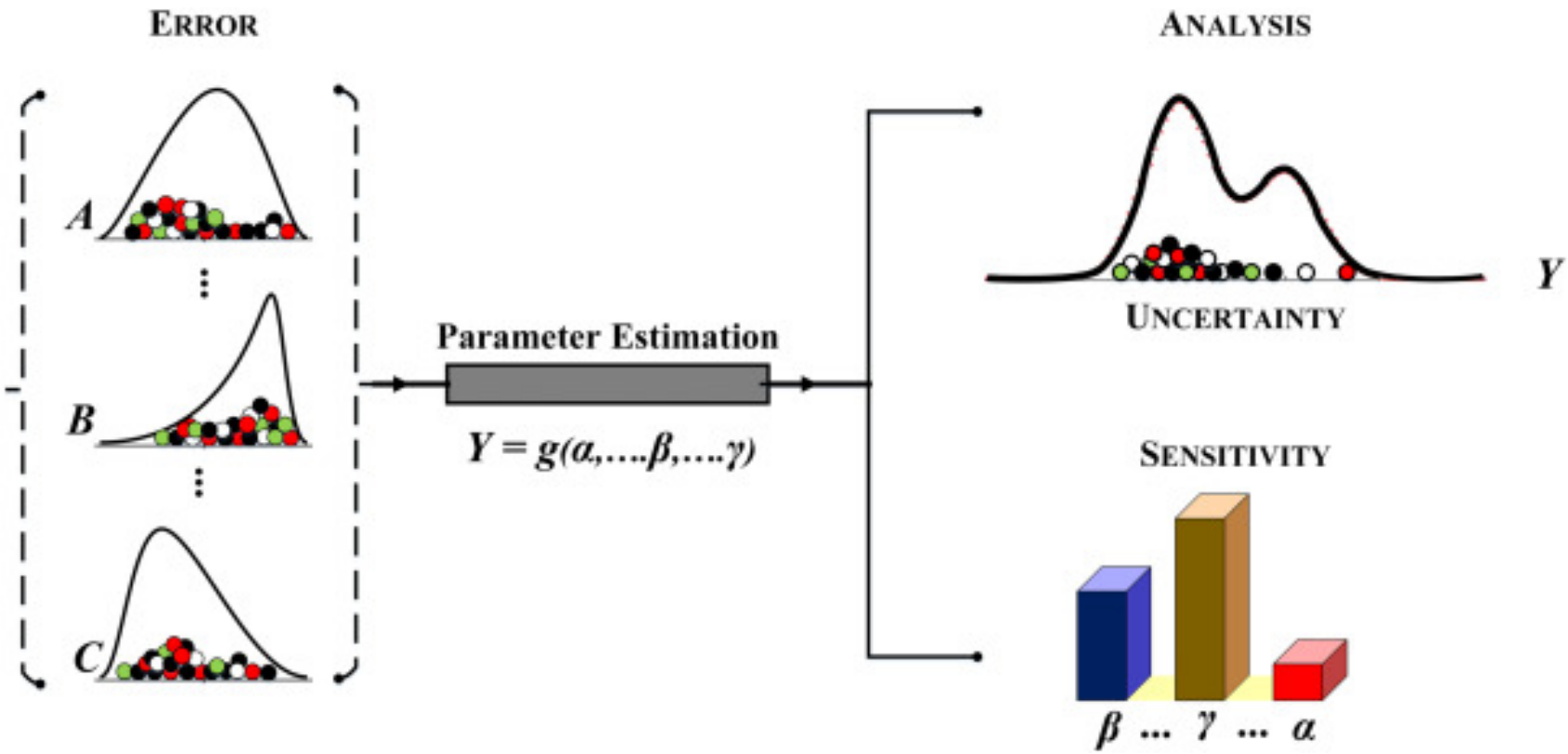


Figure 2: Nawaz, A., Arora, A. S., Yun, C. M., Lee, J. J., & Lee, M. (2021).

Purpose of Uncertainty/sensitivity analysis

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   - Evaluate how uncertainties in input parameters affect the reliability of predictions.
3. Enhancing Decision-Making Confidence:
   - Provide decision-makers with a clearer understanding of the model's behavior under different conditions.

Types of Sensitivity Analysis

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- Multi-Way Sensitivity Analysis: Examine interactions among multiple variables simultaneously to capture complex relationships.

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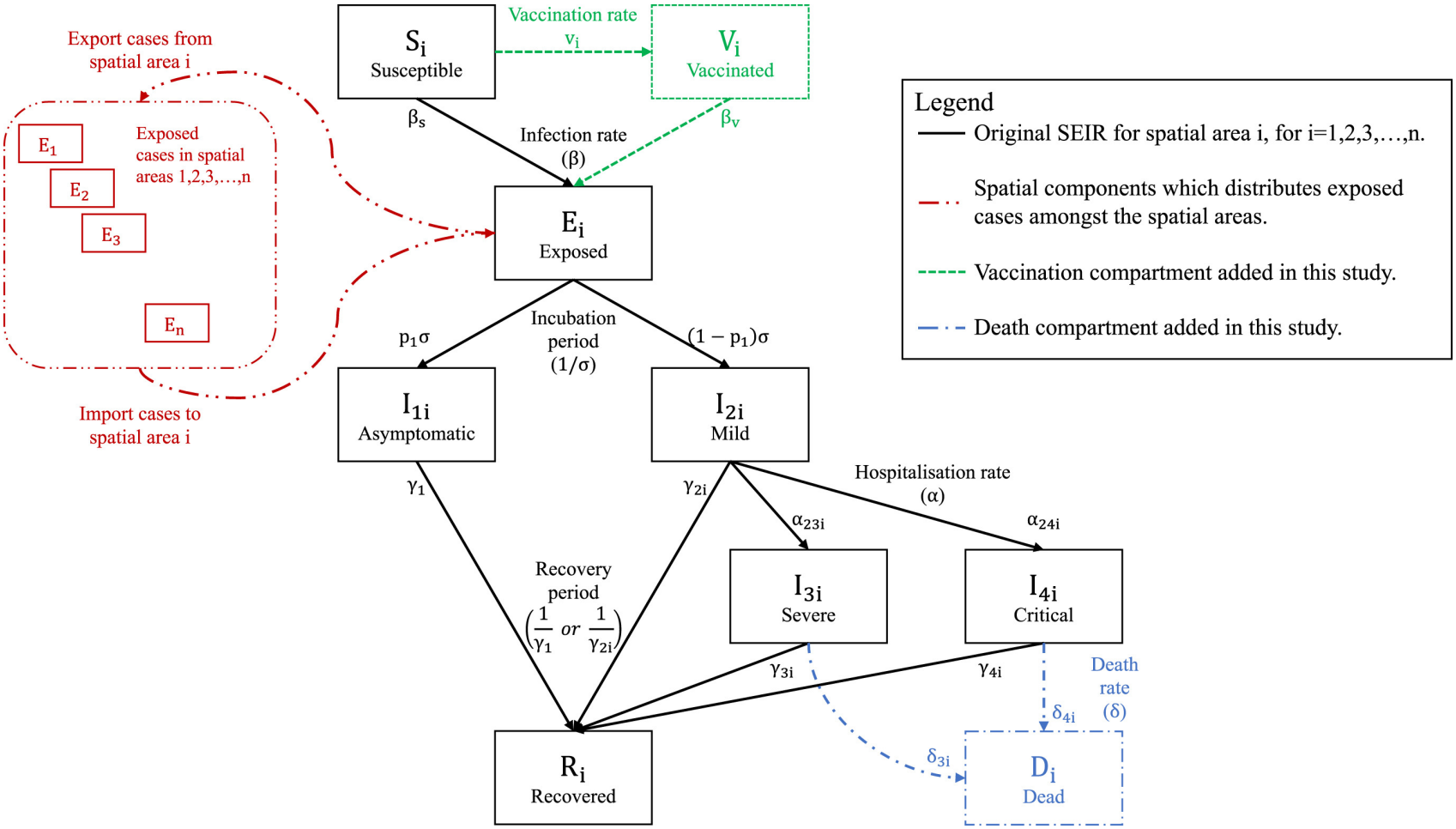
2. Identify Input Variables: List all relevant input parameters that influence the model's outcomes.
3. Determine Input Ranges: Specify the range of values for each input parameter over which the sensitivity analysis will be conducted.
4. Perform Sensitivity Analysis: Utilise appropriate mathematical or simulation techniques to analyze sensitivity.

Practical examples



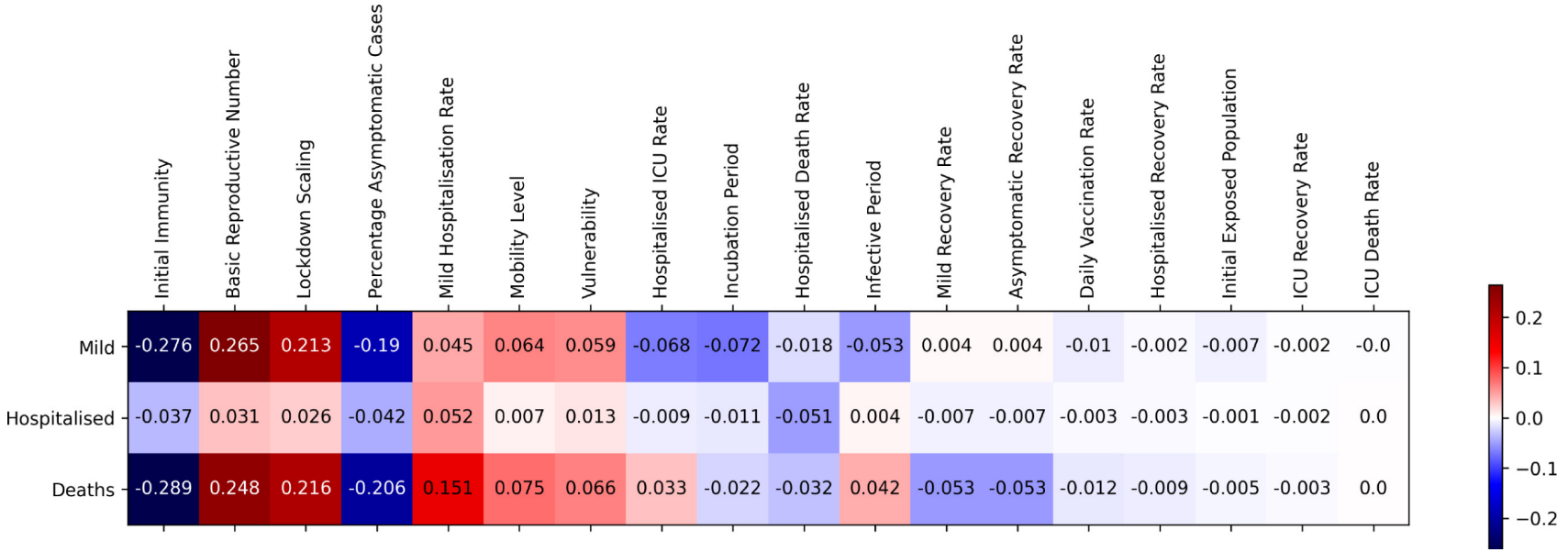


Figure 4: Dresselhaus, Claudia, et al. "A spatial model with vaccinations for COVID-19 in South Africa." Spatial Statistics (2023): 100792.

Simple exercise (as promised by Inger)

For this exercise, we will first build a very simple SIR model. This model has 3 compartments: S, I and R for the number of susceptible, infected and removed individuals. You have seen this many times!

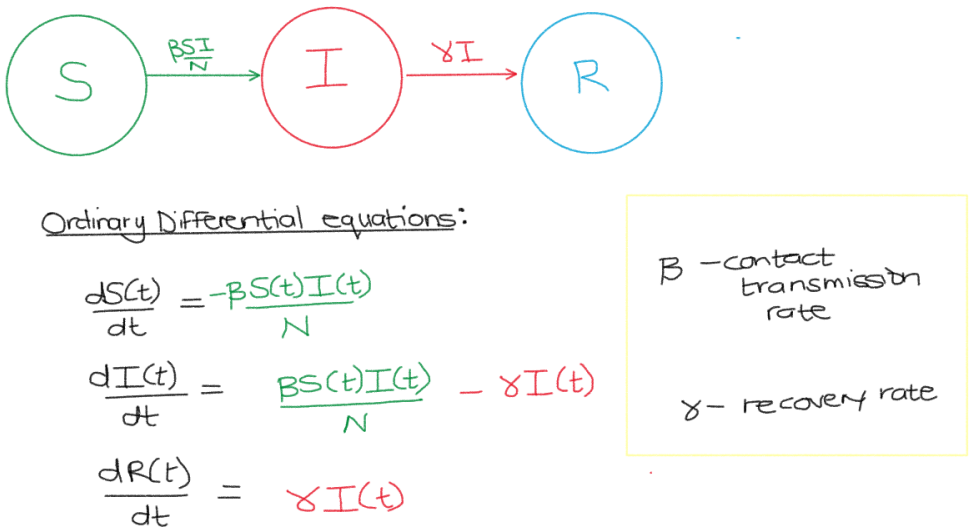
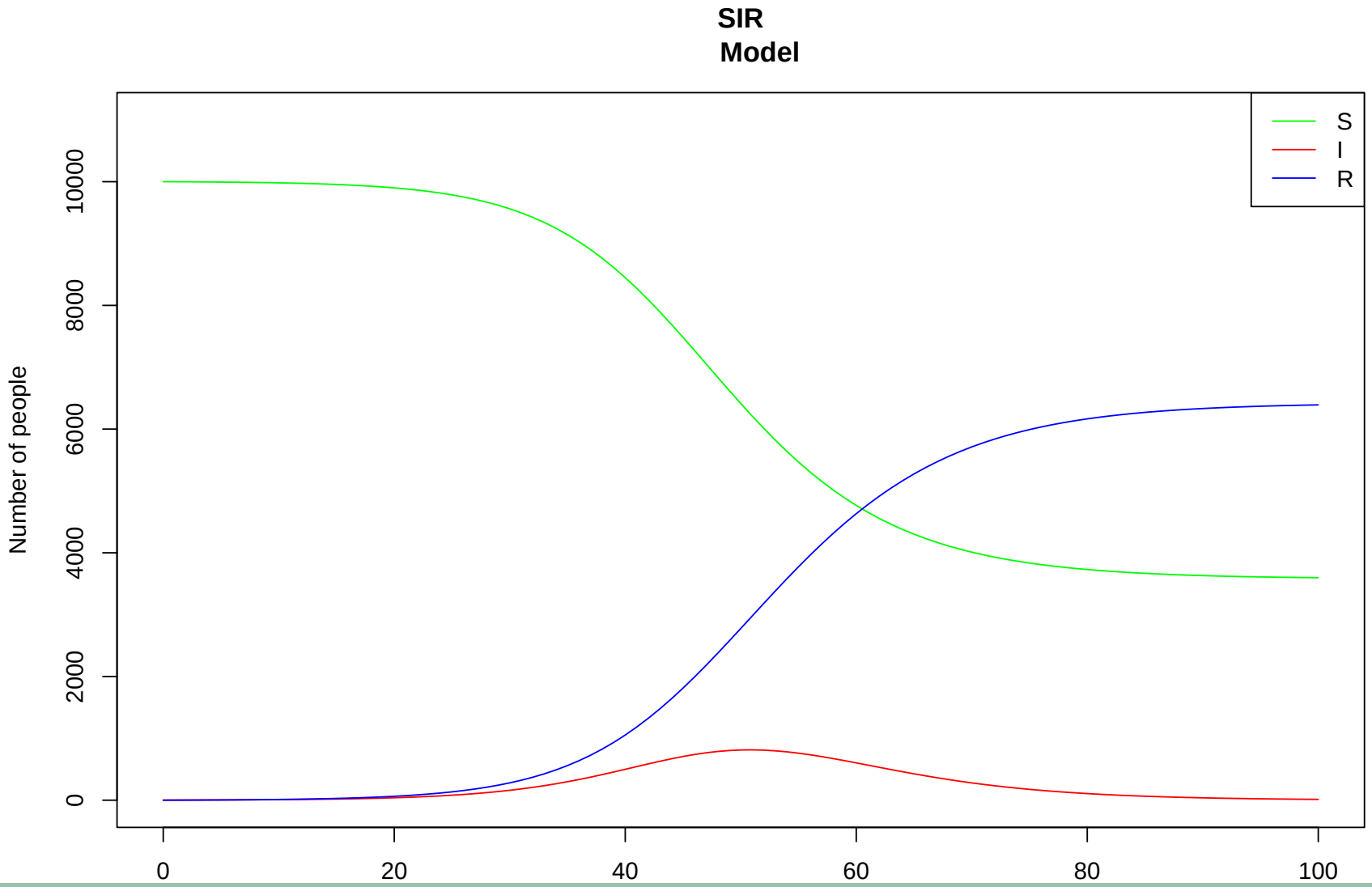


Figure 5: Dont' judge my drawing :/

The model

```
library(deSolve)
SIR <- function(t, x, parms)  {
  with(as.list(c(parms, x)), {
     N = S + I + R
    dS = -beta*(S/N)*(I)
    dI = beta*(S/N)*(I) -gamma*I
    dR = gamma*I
      output <- c(dS, dI,dR)
    list(output)
  })
}
```

continued

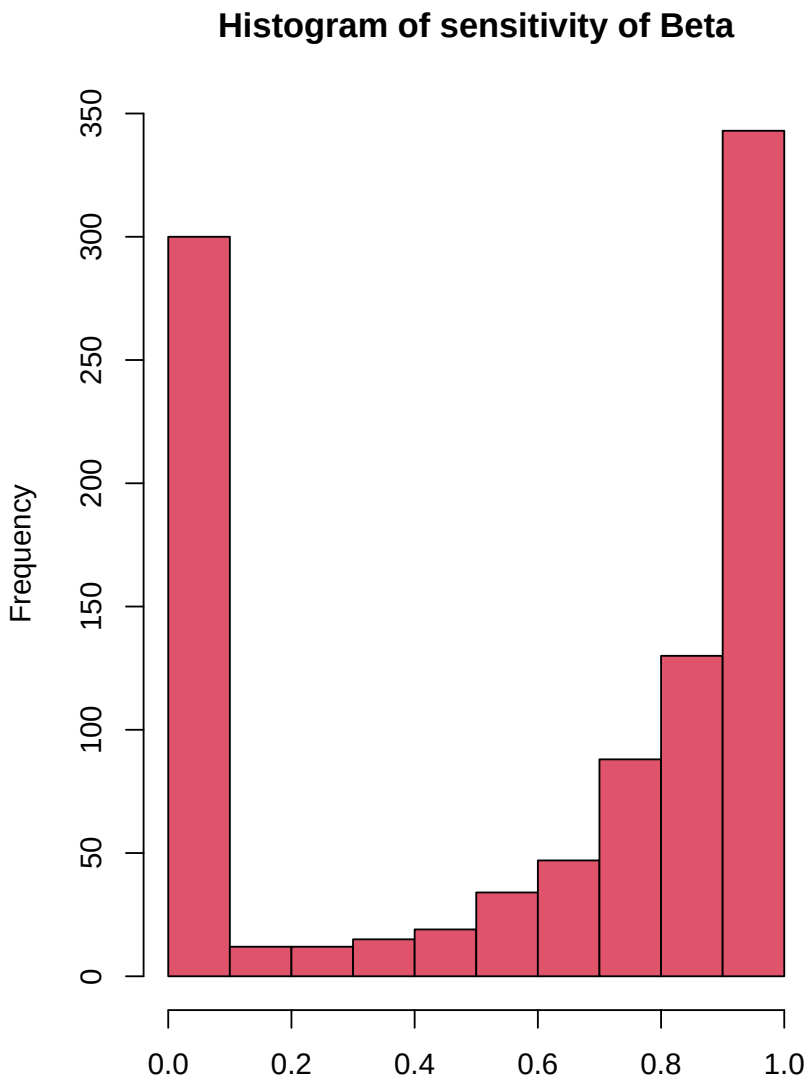

SIR
Model
S
I
R
Number of people
0
2000
4000
6000
8000
10000
0
20
40
60
80
100

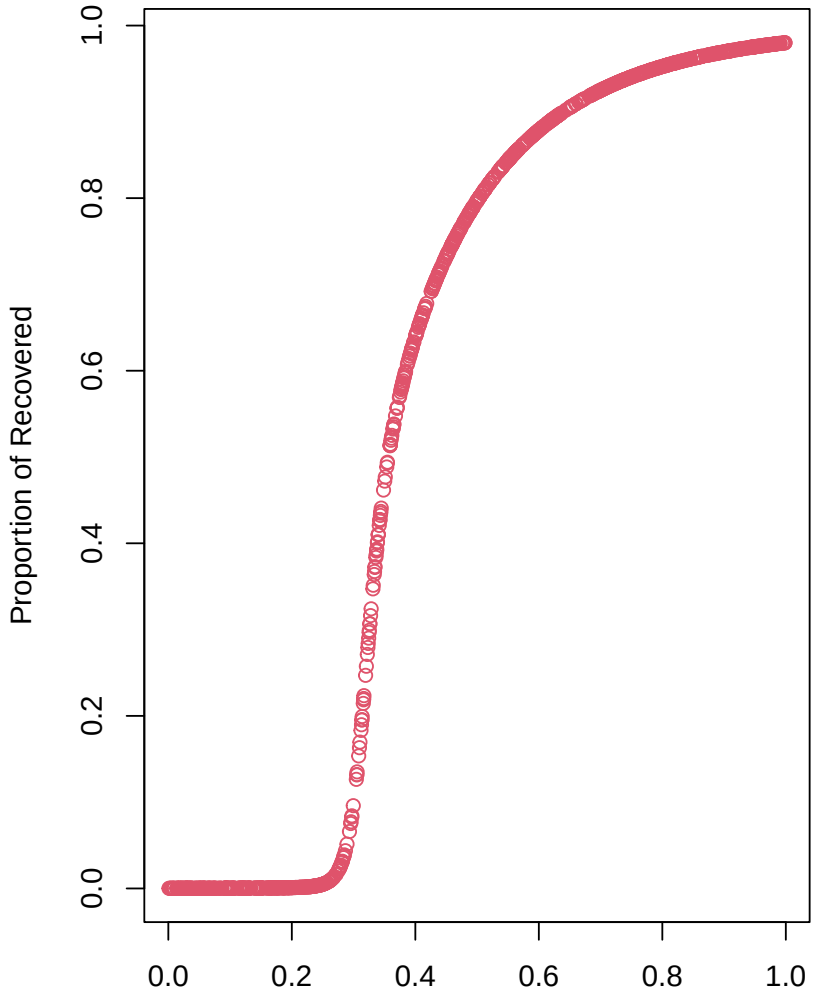
One parameter sensitivity analysis

There are only two parameters in this model, β and γ. We will start with a single parameter sensitivity of each of the parameters on the proportion of recovered individuals. You can check the sensitivity of any output variable wrt the parameters.

```
betasens<-NULL for (i in 1:1000){ parms <-
c(beta=runif(1,0,1),gamma=1/4) run_d<-ode(times=times,
y=start, func=SIR,parms=parms) betasens<-
rbind(betasens,c(parms[1],run_d[length(run_d[,1]),4]/pop))
}‘
```

Plot


Histogram of sensitivity of Beta
Frequency
0
50
100
150
200
250
300
350
0.0
0.2
0.4
0.6
0.8
1.0

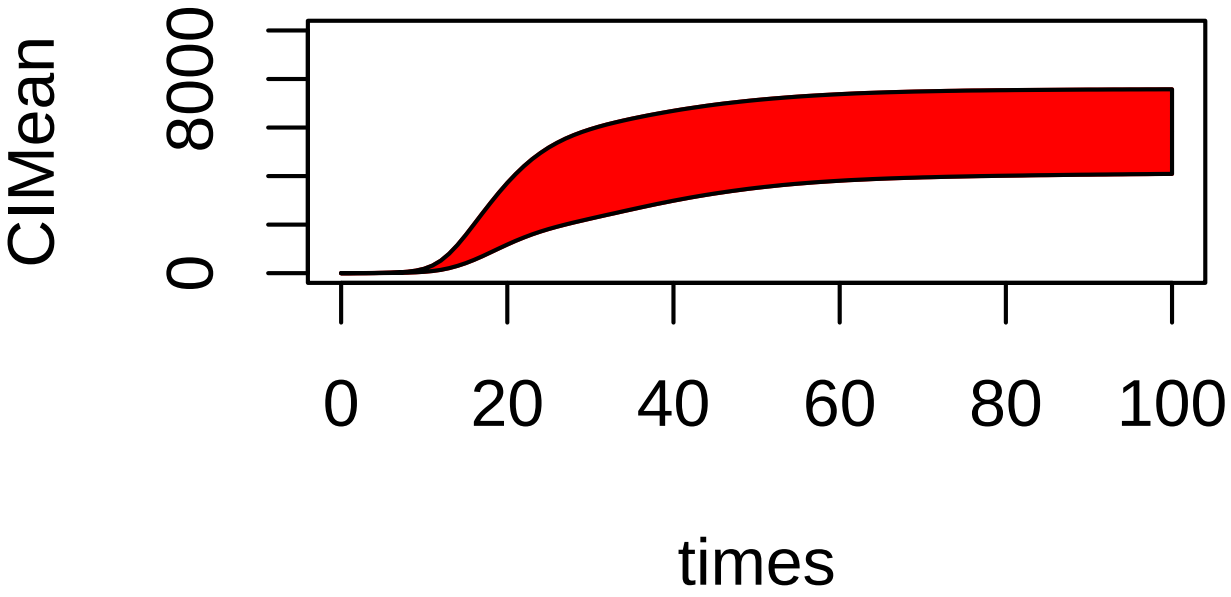

Proportion of Recovered
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0

Multivariate Sensitivity Analysis

```
CIdata<-NULL
for ( i in 1:20){
  Inc=runif(1,2,6)
  parms <- c(beta=runif(1,0,1), gamma=1/Inc)
  run_d<-ode(times=times, y=start, func=SIR,parms=parms)
  CIdata<-cbind(CIdata,run_d[,4])
}
CIsd<-CIuci<-CIlci<-NULL
CIMean<-rowMeans(CIdata)
for (i in 1:(dim(CIdata)[1])){
  CIsd[i]<-sd(CIdata[i,])
  CIuci[i]<-CIMean[i]+1.96*CIsd[i]/sqrt(20)
  CIlci[i]<-CIMean[i]-1.96*CIsd[i]/sqrt(20)
}
```

Finally

```
plot(times, CIMean, type="l", ylim=c(0,10000))
lines(times, CIlci, col="red")
lines(times, CIuci, col="red")
polygon(c(times,rev(times)), c(CIuci, rev(CIlci)), col="red
```



Concluding remarks

Figure 6: Thank you for listening