MATH 1300
How to write a matrix and other notation issues

Mathematics, much like any other language, has "typographical" as well as "grammatical" rules. The typographical rules concern notation, while the grammatical ones correspond to mathematical reasoning. This document focuses on typography, that is, it lists some common notation issues in linear algebra. It presents the accepted notation and details penalties incurred for deviating from such accepted notation.

Matrices

Consider the following objects:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{pmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{bmatrix}, \quad C = \begin{vmatrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{vmatrix}$$

$$D = \begin{matrix} 1 & 2 & 3 \\ a & 5 & b, \\ c & d & 6 \end{matrix} \quad E = \begin{pmatrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{pmatrix}, \quad F = \begin{matrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{matrix}.$$

- A and B are 3×3 matrices, which we often write $A, B \in \mathcal{M}_{33}$. They are actually equal, and both notation are accepted (try to be consistent and use always the same one, that's all).
- C is the determinant of A (and B). It is a scalar, i.e., $C \in \mathbb{R}$. We are allowed to write $C = |A| = |B|$.
- D, E and F, on the other hand, are **not** mathematical objects. They are **nothing**. As such, they should not appear anywhwere on any work you hand in, whether quizz, midterm or final. If they do appear, then you will get for the question where they do appear as much credit as they are worth: **nothing**. Regardless of whether anything else is correct.

Elementary row operations

Indicate elementary row operations to the left of the matrix that results from the operation. Two notation are accepted, with one less risky than the other. The first one indicates the operation that took place on the row that was modified:

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 3 & 1 \end{bmatrix} \quad R_2 - R_1 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

The second proceeds is essentially the same but indicates which row was affected:

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 3 & 1 \end{bmatrix} \quad R_2 \leftarrow R_2 - R_1 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

The second notation is less prone to error than the first. Any other notation will result in points being deduced.

Equal signs

The **equal sign** is a very powerful symbol in mathematics. It signifies that what is on the left is exactly the same as what is on the right. So the following statement

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 3 & 1 \end{bmatrix} \quad = R_2 \leftarrow R_2 - R_1 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

is wrong. Similarly, if

$$A = \begin{pmatrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{pmatrix}, \quad C = \begin{vmatrix} 1 & 2 & 3 \\ a & 5 & b \\ c & d & 6 \end{vmatrix}$$

then do not write $C = A$: it would be illegal since $C \in \mathbb{R}$ and $A \in \mathcal{M}_{33}$.

In this course, we manipulate a lot of objects of different sizes and shapes (vectors, matrices, scalars, etc.). Only use an equal sign if the objects on both sides of the sign are indeed equal. Otherwise, you will loose points.

Mathematics is written much like English

English is written from left to right. When the end of a line is reached, one starts the process over at the left end of the next line. Bearing this in mind, and knowing that mathematics follows essentially the same rules as English, which do you think is the acceptable way of writing the sequence of equalities $A_1 = A_2 = \cdots = A_n$?

$$\begin{array}{llll} A_1 & = A_5 & \cdots & = A_{n-3} \\ = A_2 & = A_6 & & = A_{n-2} \\ = A_3 & = A_7 & & = A_{n-1} \\ = A_4 & = A_8 & & = A_n \end{array}$$

or

$$\begin{array}{c} A_1 = A_2 = A_3 = A_4 \\ = A_5 = A_6 = A_7 = A_8 \\ = \cdots \\ = A_{n-3} = A_{n-2} = A_{n-1} = A_n \end{array}$$

If you chose the first way, then you are going to loose some points.