## A gentle introduction to Matlab

The "Mat" in Matlab does not stand for "mathematics", but for "matrix"..
$\Rightarrow$ all objects in matlab are matrices of some sort! Keep this in mind when using it.

Matlab is a high level interpreted programming language:

- a matlab program is typically a set of instructions that are evaluated iteratively;
- most of the work can be done directly from the command line.


## Defining a function

We want to plot the iterates of some function $f$. First, we define the function.
>> f=inline('r.*x.*(1-x)','x','r')
f =

$$
\begin{aligned}
& \text { Inline function: } \\
& \mathrm{f}(\mathrm{x}, \mathrm{r})=\mathrm{r} . * \mathrm{x} . *(1-\mathrm{x})
\end{aligned}
$$

This defines a function (here, with two arguments, $x$ and $r$ ), that can then be used:
>> $f(0.2,3.2)$
ans $=$
0.5120

## ";" hides the result on the command line

Remark that<br>>> $f(0.2,3.2)$<br>ans $=$<br>0.5120<br>but<br>>> f(0.2,3.2);<br>produces no output.

## Creating a vector

To create a vector, use the command

$$
x=\text { first entry }: \text { step : last entry }
$$

or, if entries are a subset of the integers,

$$
x=\text { first entry }: \text { last entry }
$$

For example, we want to plot the iterates of the logistic map, so $\mathrm{x}=0: 0.01: 1$;

Note the ";": otherwise, we get the full 101 elements vector displayed.

## What is the size of .. ?

As mentioned, in matlab everything is a matrix. For matrix operations, size is important, and it is frequent to make mistakes. To check, whos and size. whos gives a lot of information.
>> whos x

| Name | Size | Bytes | Class |  |
| :---: | :---: | :---: | :---: | :---: |
| X | 1x101 | 808 | double | array |
| Grand total is 101 elements using 808 bytes |  |  |  |  |
| Various variables can be listed on the line after whos: |  |  |  |  |
| >> whos x k |  |  |  |  |
| Name | Size | Bytes | Class |  |
| k | 1x1 | 8 | double | array |
| x | 1x101 | 808 | double | array |
| Grand total is 102 elements using 816 bytes |  |  |  |  |

## size

size, on the other hand, is "attributable". It can be used like this
>> size(x)
ans $=$
1101
but also like this, since the result is a vector
>> [r, c]=size(x)
r =
1
c = 101
in which case, $r$ and $c$ take the values of the numbers of rows and columns, respectively.

## Vectorized functions versus nonvectorized functions

Recall that we wrote
>> f=inline('r.*x.*(1-x)','x','r')
that is, every multiplication sign took the form .* instead of $*$. Here, this is needed: we want to use the vectorized form of the function, and be able to pass to $f$ a vector instead of a single value. The .* form means that the operation is applied to every entry in the vector/matrix. Same exists for / and . Can also use the function vectorize.

The result of using this vectorized form is that $f$ will be applied to every entry of $x$, and will produce a vector.

Vectorized operations have been optimized in matlab, and are extremely fast. When possible, they should be used instead of loops.

## Vectorized vs nonvectorized

Define
>> f=inline('r.*x.*(1-x)','x', 'r')
>> g=inline('r*x*(1-x)', 'x', 'r')
and for simplicity, consider the vector
>> $x=[1,2]$;
Then
>> f(x,3.5)
$g(x, 3.5)$
ans $=$
$0 \quad-7$
??? Error using ==> inlineeval
Error in inline expression ==> r*x*(1-x)
??? Error using ==> mtimes
Inner matrix dimensions must agree.

## Plotting

Basic plotting is very easy. The format is
plot(x_axis,y_value)
so, for example (with $f$ as defined above),
plot( $x, f(x, 3.4)$ )
(here, ";" or not does not matter, as the figure appears in a new window and all that ";" changes is the output in the command window).


## Making things a bit more fancy

This is a very basic plot.

- We could want to plot more than one object (for example, the line $y=x$ would be nice)..
plot( $x, x, x, f(x, 3.4)$ );
Ordering is by pairs: $x_{1}, f_{1}\left(x_{1}\right), x_{2}, f_{2}\left(x_{2}\right)$. Two elements in a pair must have the same number of columns. Different pairs can have different numbers of columns. Each element in a given pair can be a point, a vector, a matrix.
- We could want to label the axes..

```
xlabel('x');
ylabel('f(x)');
```



## Computing several iterates

For the moment, we only have $f(x)$. We want $f^{n}(x)$, for a given $n$. Several ways.

- Taking for example $r=3.5$, use $\mathrm{f}(\mathrm{f}(\mathrm{x}, 3.5), 3.5)$
- The downside to this method is that matlab does not allow to formally define $f^{n}$, so tricks have to be used for larger values of $n$, for example, produce a string containing the command $\mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{f}(\mathrm{x}, 3.5), 3.5), 3.5), 3.5), 3.5)$ and evaluate it. Complicated..
- Another method consists in using the result found at the previous step to evaluate the next. We do that..


## Automatic resizing of vectors and matrices

We are going to use a very nice feature of matlab: adding elements to a vector, or rows/columns to a matrix, is automatic. Suppose for example that we had defined $x$ as
$\mathrm{x}=0: 0.01: 0.5$;
Then
$x=[x, 0.51: 0.01: 1] ;$
would produce the vector $x$ as we had earlier.

Be careful! Note that the command was
$x=[x, 0.51: 0.01: 1]$;
that is, the old and new entries were separated by a ",". This is horizontal concatenation. The command with a ";" tries to add a new row. In our case, we get
>> z=[z;0.51:0.01:1]
??? Error using ==> vertcat
All rows in the bracketed expression must have the same number of columns.
because we are trying to add a row of 50 elements to a row of 51 elements. But
>> $z=[z ; 0.51: 0.01: 1.01]$
works, and gives a $2 \times 51$ matrix.

Here, we are going to use the latter form of the command, and add each successive iterate to a solution matrix $M$.
First, define an empty matrix,
$\mathrm{M}=[]$;
Then we need to loop from 1 to $n$, where $n$ is the iterate that we want.

## Loops

The command uses the same type of syntax as the creation of a vector: to loop from 4 to 12 by steps of 1 ,
for $i=4: 12$, command(s) to be repeated, maybe using the value i end;
whereas to loop by non-unit or non-integer steps, say from 4 to 12 by steps of 1.35 ,
for $i=4: 1.35: 12$,
command(s) to be repeated, maybe using the value i end;

Note that in that case, the last $i$ is equal to 10.75 , not 12 , since $10.75+1.35=12.1>12$. The same is true when using non-unit steps to create vectors.

## Accessing matrix elements

Suppose that $M$ is an $m \times n$-matrix. Then

- $M(i, j)$ is the element on the $i$ th row and $j$ th column.
- M(i,:) is the $i$ th row.
- $M(:, j)$ is the $j$ th column.
- $M$ (end,:) is the last row of $M$ (end is a reserved word which always points to the last valid index in a given matrix dimension).
- $M(:$, end $)$ is the last column of $M$.
- $M$ (end,1:10) are the first 10 entries in the last row of $M$.
- $M(1: 2,3: 5)$ is the submatrix of $M$ consisting of rows 1 and 2 and columns 3 to 5 of $M$.


## Back to the iterates

After some thought, we realize that we will need to go back one iterate. So instead of starting with empty matrix $M$, fill the first row of $M$ with first iterate, and start at iterate 2.

```
n=10;
r=3.5;
M=f(x,r);
for i=2:n,
    M=[M;f(M(end,:),r)];
end;
plot(x,M);
```

This plots all the iterates to $n$. A bit crowded..


