Water-borne and environmentally-transmitted diseases
MATH 8xyz – Lecture 11

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Outline

A model for cholera

Last remarks

Research article

Endemic and epidemic dynamics of cholera: the role of the aquatic reservoir
Cláudia Torres Codeço*
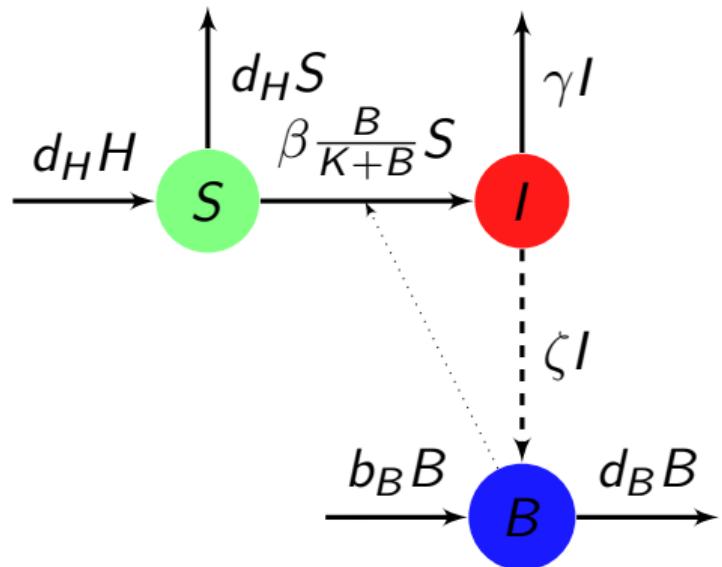
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Codeço's model
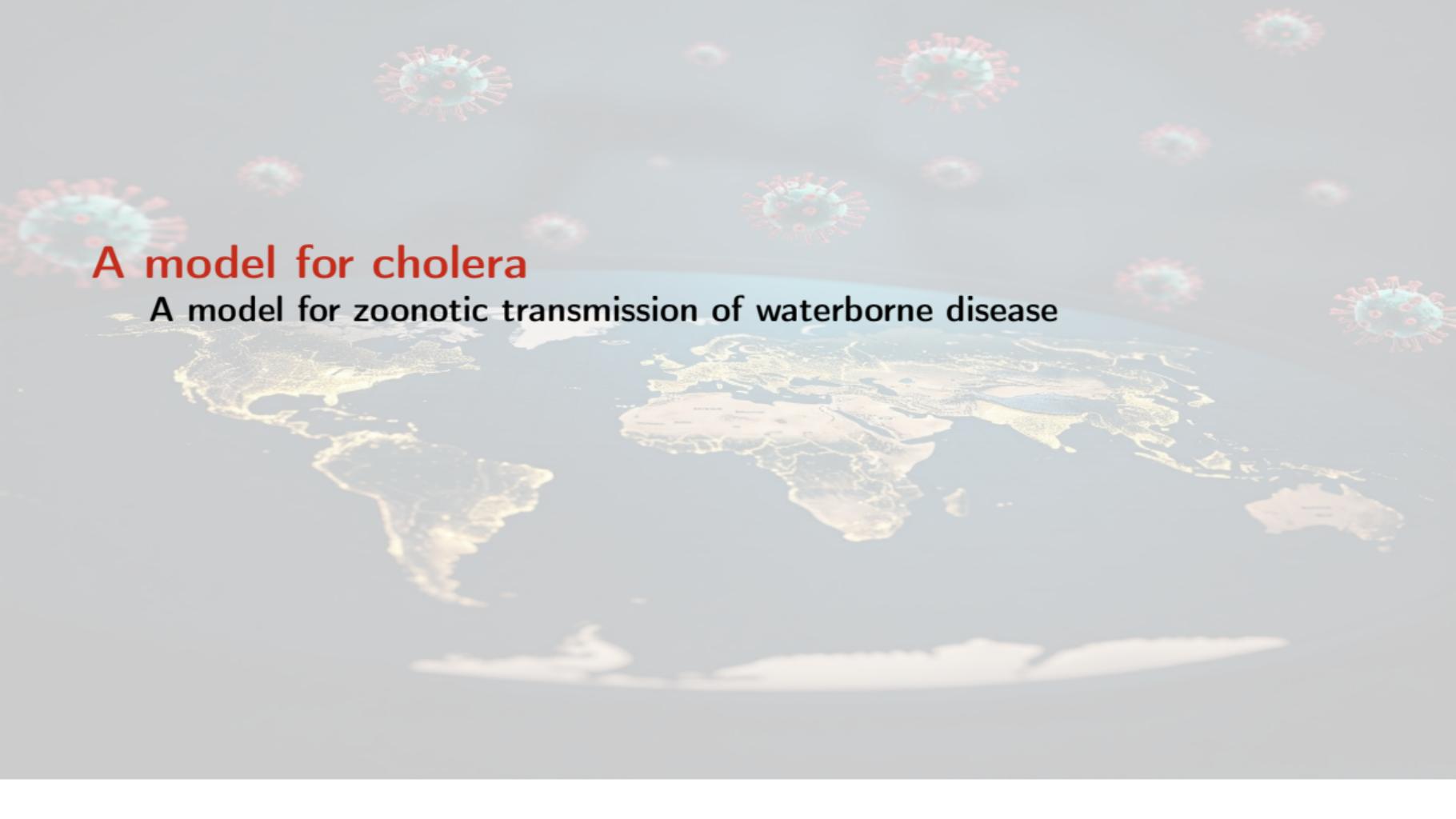
$$S' = d_H(H - S) - \beta \frac{B}{K + B} S \qquad (1a)$$

$$I' = \beta \frac{B}{K + B} S - \gamma I \qquad (1b)$$

$$B' = (b_B - d_B)B + \zeta I \qquad (1c)$$

K concentration of cholera in water giving 50% chance of catching it

Note that the dashed arrow from I to B is not a flow: individuals do not convert into *vibrio cholerae*

A model for cholera

A model for zoonotic transmission of waterborne disease
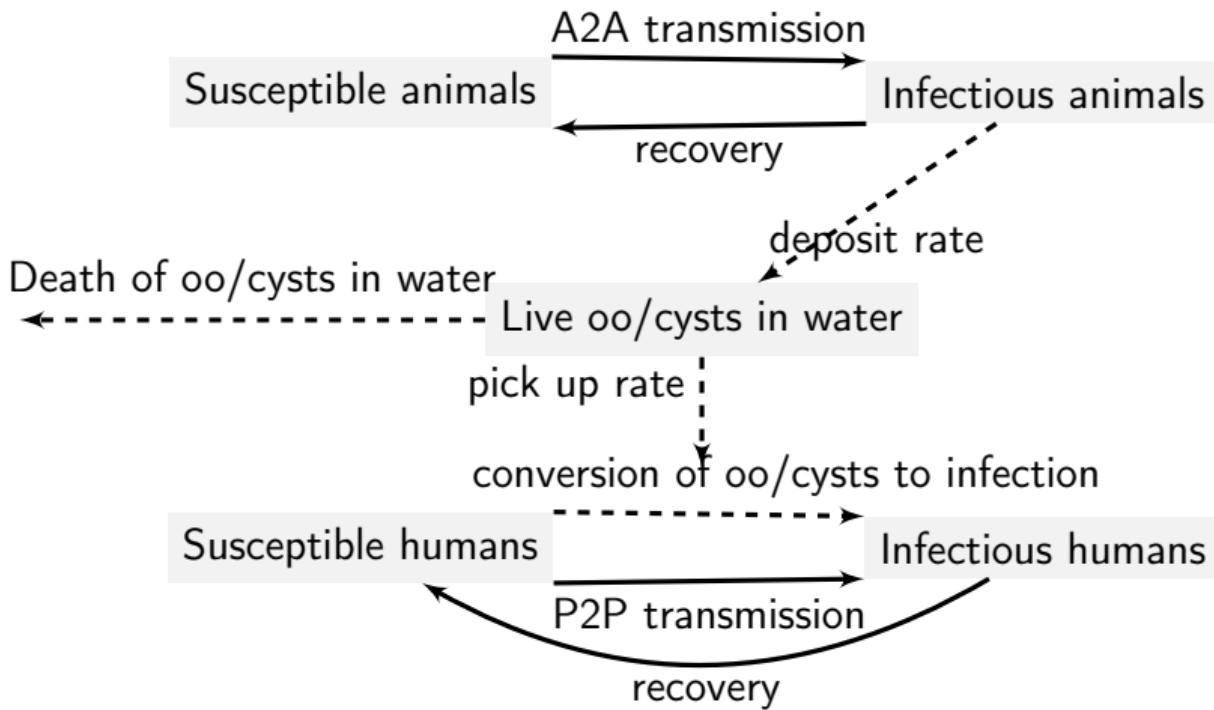
Zoonotic Transmission of Waterborne Disease:
A Mathematical Model
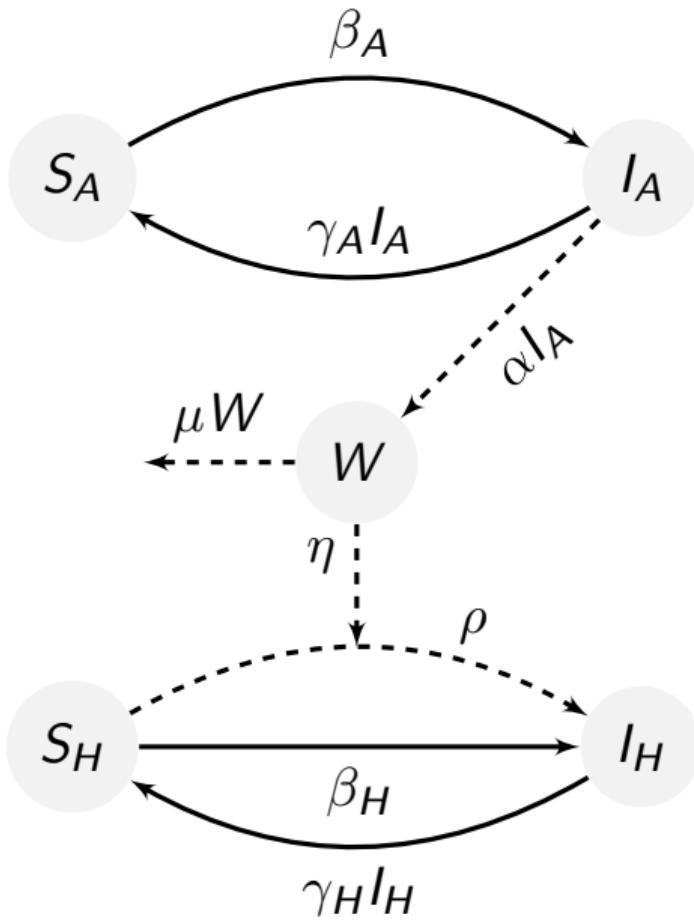
Edward K. Waters[1] · Andrew J. Hamilton[2] ·
Harvinder S. Sidhu[3] · Leesa A. Sidhu[3] ·
Michelle Dunbar[4]

Zoonotic transmission of waterborne disease

Zoonoses are animal diseases that are transmitted to humans

Model here used for instance to model Giardia transmission from possums to humans

A2A transmission

Susceptible animals → Infectious animals

recovery

deposit rate

Death of oo/cysts in water ← Live oo/cysts in water

pick up rate

conversion of oo/cysts to infection

Susceptible humans → Infectious humans

P2P transmission

recovery

The full model

$$S_A{}' = -\beta_A S_A I_A + \gamma_A I_A \tag{2a}$$
$$I_A{}' = \beta_A S_A I_A - \gamma_A I_A \tag{2b}$$
$$W' = \alpha I_A - \eta W(S_H + I_H) - \mu W \tag{2c}$$
$$S_H{}' = -\rho \eta W S_H - \beta_H S_H I_H + \gamma_H I_H \tag{2d}$$
$$I_H{}' = \rho \eta W S_H + \beta_H S_H I_H - \gamma_H I_H \tag{2e}$$

Considered with $N_A = S_A + I_A$ and $N_H = S_H + I_H$ constant

Simplified model

Because N_A and N_H are constant, (2) can be simplified:

$$I_A' = \beta_A N_A I_A - \gamma_A I_A - \beta_A I_A^2 \tag{3a}$$

$$W' = \alpha I_A - \eta W N_H - \mu W \tag{3b}$$

$$I_H' = \rho \eta W (N_H - I_H) + \beta_H N_H I_H - \gamma_H I_H - \beta_H I_H^2 \tag{3c}$$

Three EP: DFE $(0, 0, 0)$; endemic disease in humans because of H2H transmission; endemic in both H and A because of W
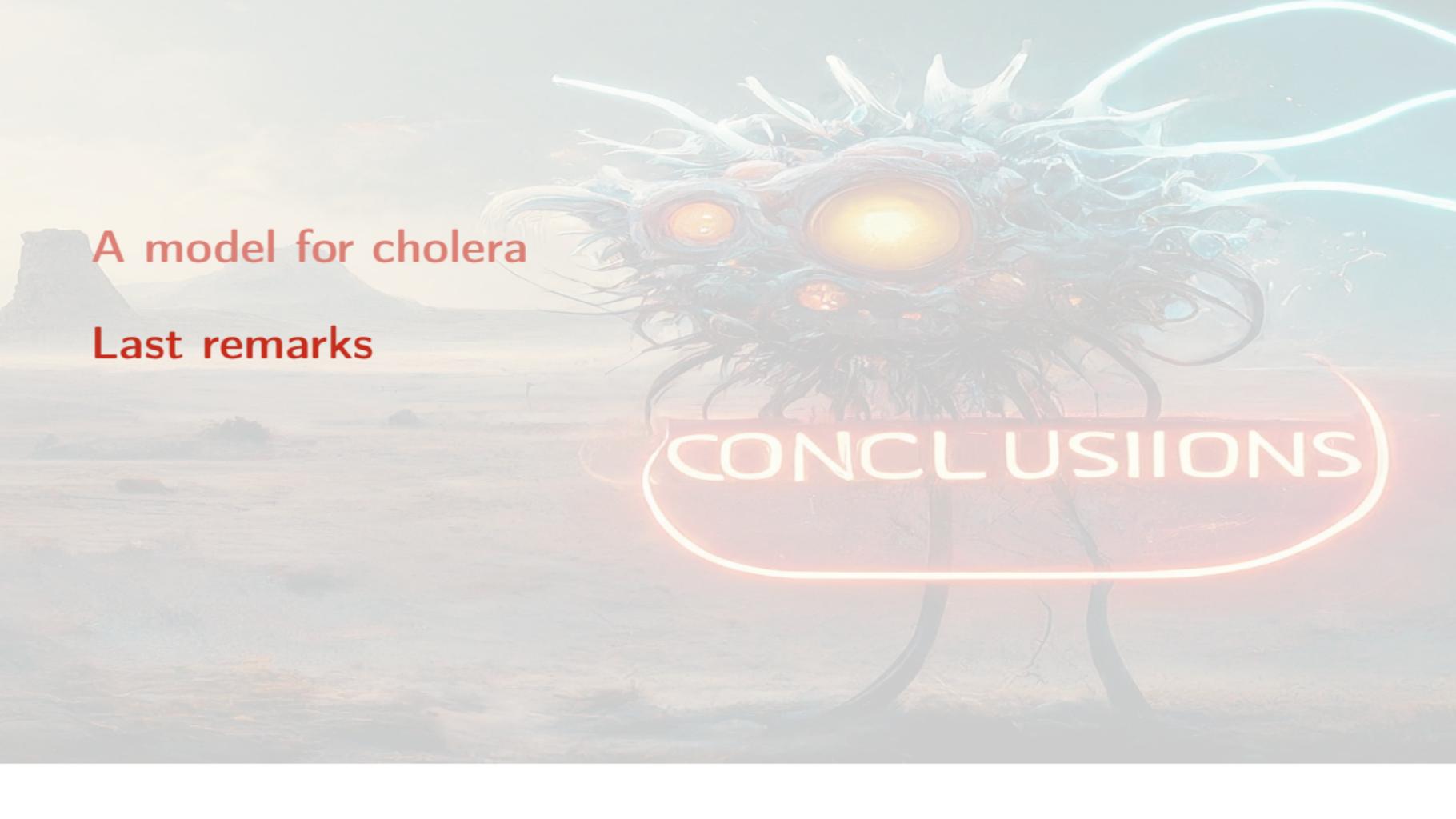
Three EP: DFE $(0, 0, 0)$; endemic disease in humans because of H2H transmission; endemic in both H and A because of W

Let

$$\mathcal{R}_{0A} = \frac{\beta_A}{\gamma_A} N_A \quad \text{and} \quad \mathcal{R}_{0H} = \frac{\beta_H}{\gamma_H} N_H \tag{4}$$

▶ DFE LAS if $\mathcal{R}_{0A} < 1$ and $\mathcal{R}_{0H} < 1$, unstable if $\mathcal{R}_{0A} > 1$ or $\mathcal{R}_{0H} > 1$

▶ If $\mathcal{R}_{0H} > 1$ and $\mathcal{R}_{0A} < 1$, (3) goes to EP with endemicity only in humans

▶ Endemic EP with both A and H requires $\mathcal{R}_{0A} > 1$ and $\mathcal{R}_{0H} < 1$

Note that proof is **not** global

A model for cholera

Last remarks

CONCL USIIONS

To simplify or not to simplify?

▶ In the KMK epidemic model (**??**) and the SIRS endemic model (**??**), since the total population is constant or asymptotically constant, it is possible to omit one of the state variables since $N^\star = S + I + R$

▶ We often use $R = N^\star - S - I$

▶ This can greatly simplify some computations

▶ Whether to do it or not is a matter of preference

To normalise or not to normalise?

▶ In the KMK epidemic model (**??**) and the SIRS endemic model (**??**), since the total population is constant or asymptotically constant, it is possible to normalise to $N = 1$

▶ This can greatly simplify some computations

▶ However, I am not a big fan: it is important to always have the "sizes" of objects in mind

▶ If you do normalise, at least for a paper destined to mathematical biology, always do a "return to biology", i.e., interpret your results in a biological light, which often implies to return to original values

Where we are

▶ An *epidemic* SIR model (the KMK SIR) in which the presence or absence of an epidemic wave is characterised by the value of $\mathcal{R}_0$

▶ The KMK SIR has explicit solutions (in some sense). **This is an exception!**

▶ An *endemic* SIRS model in which the threshold $\mathcal{R}_0 = 1$ is such that, when $\mathcal{R}_0 < 1$, the disease goes extinct, whereas when $\mathcal{R}_0 > 1$, the disease becomes established in the population

▶ Some simple variations on these models

▶ A few models for vector-borne or water-borne diseases

Bibliography I